Challenge 3 (pt. 2), due next Wednesday in class

In this challenge, I plan to help you understand the underlying intuition behind real options theory. The idea here is closely connected to option pricing models we'll cover, although real options analysis really took off after options pricing models were formally developed. My main goal with challenge 3 is for you to understand that while exchange-traded options are what most people have in mind when they refer to options, option-like investments or situations can be applied much more broadly. The key to options is the insight that you own the right to do something, but not the obligation to do so.

You own a large quantity of corn. Today ($t = 0$), this corn has a certain grade, or quality, q^0. The price per bushel of corn is given by p. As we've seen before, revenue is given by price times quantity. Assume the latter to be 1. In practice, because the better the grade, the higher the price you get for corn, we can consider revenue to be simply pq^0. Corn might go bad, just like homes or capital depreciate. Spoilage rate is given by $\delta \geq 0$. We'll consider corn as an asset so that there's some future stream of revenue we get while corn is still under storage (maybe people like to visit your warehouse and take pictures of the corn, and you charge them the market value of corn pq^0). The discount (interest) rate is r. **Explain why the formula below gives the present value of corn in this case** (we assume corn can last forever, because of Ceres' blessings):

$$PV(q^0) = \int_0^\infty (pq^0)e^{-(\delta+r)t}dt$$

After doing that, give some intuition on how quality q^0 depends on spoilage rate. Is the grade of corn always going to be the same? Why?

You now realize that during all this time a corn-embellishment product was available. This product turns corn into a green-like grain (perhaps for St. Patrick's?), but after crushed or processed, the corn comes back to the normal yellow. This treatment actually reflects a one-time call option: you always had the right to exercise it (turn the corn into green). If you exercise the option, two things happen: first, you have to pay to have the treatment, which costs k. Second, you can never come back to the yellow corn (unless after sold and processed). Because people visiting the warehouse really love the green effect, the "quality" of corn increases from q^0 to q^{green} after the treatment. However, potential corn buyers might be scared of the color and not believe the final

processed product will indeed turn out yellow. Thus, q^{green} could also be smaller than q^0.

If you choose to exercise the option, you do so in t^*. **Explain why the equation below gives the value of corn, including the option to "upgrade" it:**

$$vq^0 + \max\{v(q^{green} - q^0 e^{-(\delta)t^*} - k), 0\}e^{-(r)t^*}$$

where v is the value of corn given by the first equation. Don't focus too much on what the exponential operators imply. I want you to explain to me what the first and second terms mean.