Lecture XIII

ACE 428
Commodity Futures and Options

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Options Pricing

Synthetic positions

- You can create synthetic futures positions with options
- Combined payoff of your options positions is the same (or nearly the same) as the payoff of an outright futures position
- Synthetic long futures
  - long call + short put (same strike)
- Synthetic short futures
  - long put + short call (same strike)

Synthetic positions (cont.)

- Example: create a long position on wheat futures
- Wheat market on 03/06/2018
  - futures price for December'18 = \$5.38/bu
  - Dec'18 call (go long)
    - strike = \$5.20/bu
    - premium = \$0.60/bu
  - Dec'18 put (go short)
    - strike = \$5.20/bu
    - premium = \$0.17/bu

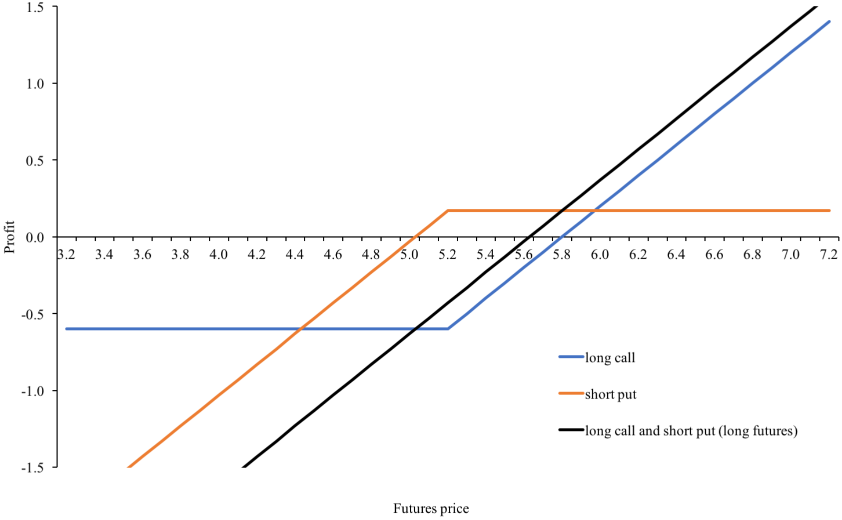
Synthetic positions (cont.)

- If underlying futures price goes above \$5.20/bu
  - you exercise your call and gain the difference between futures price and strike price
  - the put you sold is not exercised
- underlying futures price goes below \$5.20/bu
  - you don't want to exercise your call
  - the put you sold is exercised against you, and you lose the difference between strike price and futures price
- Therefore you gain (lose) if futures price goes above (below) \$5.20/bu

Synthetic positions (cont.)

- Does it look like a long futures position at \$5.20/bu?
- Yes, but here there is a cost when you build this synthetic long futures position
  - you receive only \$0.17/bu when you sell the put and pay \$0.60/bu to buy the call
  - extra cost of \$0.43/bu (0.17-0.60)
- If you take into account this cost, it's like going long at \$5.63/bu in the futures market
  - synthetic futures price = \$5.63/bu

Synthetic positions (cont.)



Synthetic positions (cont.)

- Synthetic long futures position

+ strike (K)	+5.20
− put premium (P)	−0.17
+ call premium (C)	+0.60
= synthetic futures price (F_s)	=5.63

- You make money in this synthetic long futures position as long as current futures price is above the synthetic futures price
  - current futures price ¿ synthetic futures price
- Or, you make money in this synthetic futures position as long as:
  - current futures price ¿ strike − put premium + call premium
- $F_t > K - P + C$ or $P - C > K - F_t$

Synthetic positions (cont.)

- Example: create a short position on wheat futures
- Wheat market on 03/06/2018
  - futures price for December'08 = \$5.38/bu
  - Dec'18 call (go short)
    - strike = \$5.20/bu
    - premium = \$0.60/bu
  - Dec'18 put (go long)
    - strike = \$5.20/bu
    - premium = \$0.17/bu

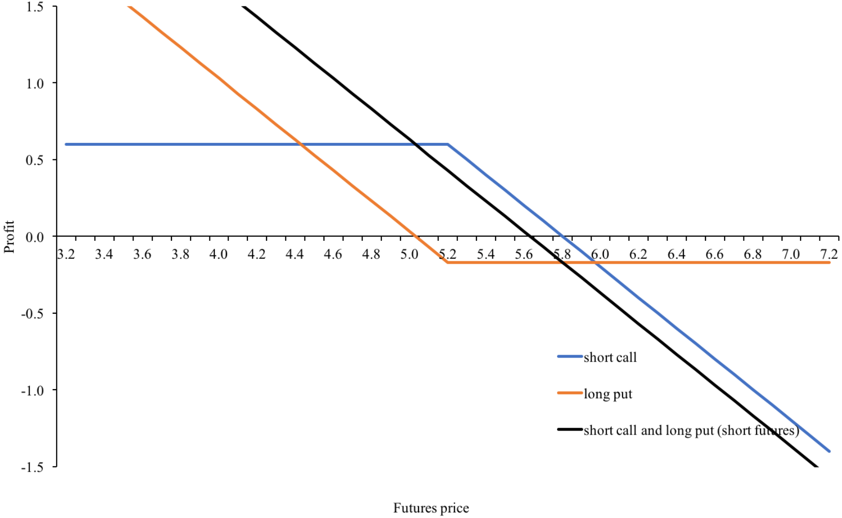
Synthetic positions (cont.)

- If underlying futures price goes above \$5.20/bu
  - you don't want to exercise your put
  - the call you sold is exercised against you, and you lose the difference between futures price and strike price
- If underlying futures price goes below \$5.20/bu
  - you exercise your put and gain the difference between strike price and futures price
  - the call you sold is not exercised
- Therefore you gain (lose) if futures price goes below (above) \$5.20/bu

Synthetic positions (cont.)

- Does it look like a short futures position at \$5.20/bu?
- Yes, but there is a revenue when you build this synthetic short futures position
  - you pay \$0.17/bu to buy the put and receive \$0.60/bu when you sell the call
  - extra revenue of \$0.43/bu (0.60-0.17)
- If you take into account this revenue, it's like going short at \$5.63/bu in the futures market
  - synthetic futures price = \$5.63/bu

Synthetic positions (cont.)



8.1 Synthetic positions (cont.)

- Synthetic short futures position

+ strike (K)	+5.20
− put premium (P)	−0.17
+ call premium (C)	+0.60
= synthetic futures price (F_s)	=5.63

- You make money in this synthetic long futures position as long as synthetic futures price is above the current futures price
  - synthetic futures price ¿ current futures price
- Or, you make money in this synthetic futures position as long as:
  - strike − put premium + call premium ¿ current futures price
- $K - P + C > F_t$ or $P - C < K - F_t$

Synthetic positions (cont.)

- You can combine futures and options contracts to create synthetic positions in both futures and options
- Examples:
  - long call + short put $\rightarrow$ long futures
  - long put + short call $\rightarrow$ short futures
  - long futures + short call $\rightarrow$ short put
  - long futures + long put $\rightarrow$ long call

Put-call parity

- Synthetic long futures is
  - profitable if $P - C > K - F_t$
- Synthetic short futures is
  - profitable if $P - C < K - F_t$
- What if equality holds?
  - no profit opportunity $P - C = K - F_t$
- So, there is an arbitrage relationship between calls and puts with same strike

Put-call parity (cont.)

- If $P - C > K - F_t$, we have $F_t > F_s$
  - traders want to arbitrage by going short the underlying futures market and long the synthetic futures market
- As traders go short the underlying futures market, F_t decreases and $(K - F_t)$ gets larger
- As traders go long the synthetic futures market,
  - they buy calls and sell puts
  - $(P - C)$ gets smaller
- Then, $(P - C)$ and $(K - F_t)$ will converge

Put-call parity (cont.)

- If $P - C < K - F_t$, we have $F_t < F_s$
  - traders want to arbitrage by going long the underlying futures market and short the synthetic futures market
- As traders go long the underlying futures market, F_t increases and $(K - F_t)$ gets smaller
- As traders go short the synthetic futures market,
  - they sell calls and buy puts
  - $(P - C)$ gets larger
- Then, $(P - C)$ and $(K - F_t)$ will converge

Put-call parity (cont.)

- In equilibrium, no arbitrage opportunities exist

$$P - C = K - F_t$$

- There is parity relationship between puts and calls known as put-call parity

$$P = C + K - F_t$$

- The specific relationship above holds for options on futures only

Put-call parity (cont.)

- Going back to our wheat example
  - P = \$0.17/bu
  - C = \$0.60/bu
  - K = \$5.20/bu
  - F_t = \$5.38/bu
- We have
  - $P - C = 0.17 - 0.60 = -0.43$
  - $K - F_t = 5.20 - 5.38 = -0.18$
  - $P - C < K - F_t$

Put-call parity (cont.)

- Put-call parity relationship allows to figure out if the spread between put and call premiums is not in equilibrium
  - arbitrage trading will drive the spread towards equilibrium
- However, this parity relationship doesn't tell you which option is mispriced
  - actually both can be mispriced
- In order to know which option is mispriced you need to be able to price options individually

Options pricing

- Prices (premiums) of options on futures are determined by two components:
  - intrinsic value, which is determined by:
    - strike price (K)
    - underlying futures price (F_t)
  - time value, which is determined by:
    - futures price volatility (σ)
    - time to maturity ($T - t$)
    - interest rates (r)

  Note: time to maturity and volatility determine the probability of a profitable move and interest rates represent the cost of money

Options pricing (cont.)

- Impact of variable changes on option premium

Variable	Variable change	Premium
Futures price	increases	increases (call)
		decreases (put)
Strike price	increases	decreases (call)
		increases (put)
Volatility	increases	increases
Time to maturity	increases	increases
Interest rate	increases	increases (call)
		decreases (put)

Options pricing (cont.)

- There are several option pricing models
- Most famous is the Black-Scholes model
  - published in 1973
  - initially developed to price options on equities
  - Black, F. and M. Scholes (1973). "The Pricing of Options and Corporate Liabilities", *The Journal of Political Economy*, 81(3):637-654.
- Following the same principles, Black developed a model for pricing options on futures
  - published in 1976
  - Black, F. (1976). "The Pricing of Commodity Contracts", *Journal of Financial Economics*, 3:167-179.

Options pricing (cont.)

- Black's model: assumptions
  - no transaction costs
  - **European option**
    - exercised only at maturity
  - interest rate is constant
  - futures prices follow a lognormal distribution
  - volatility is constant over the life of the contract

Options pricing (cont.)

- Black's model: call premium

$$C = \underbrace{e^{-r(T-t)}}_{\text{discount term}} \underbrace{\left[\underbrace{F}_{\text{futures price}} \cdot \underbrace{N(d_1)}_{\text{CDF of } d_1} - \underbrace{K}_{\text{strike price}} \cdot \underbrace{N(d_2)}_{\text{CDF of } d_2} \right]}_{\text{intrinsic value}}$$

- call premium is the present value of the intrinsic value at maturity
- $N(d_1)$ and $N(d_2)$ represent probabilities (between 0 and 1) and account for risk in this model
- $d_1 = \frac{\ln \frac{F}{K} + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = d_1 - \sigma\sqrt{T-t}$
- σ is the volatility and $T - t$ is the time to maturity

Options pricing (cont.)

- If call is deep in-the-money
  - $F > K$ and the ratio F/K is large
  - d_1 and d_2 are also large numbers
- Hence, $N(d_1)$ and $N(d_2)$ will be 1
- Call premium will be given by the intrinsic value $F - K$

$$C = \underbrace{e^{-r(T-t)}}_{\text{discount term}} \underbrace{[\underbrace{F}_{\text{futures price}} \cdot \underbrace{N(d_1)}_{=1} - \underbrace{K}_{\text{strike price}} \cdot \underbrace{N(d_2)}_{=1}]}_{\text{intrinsic value}}$$

$$= e^{-r(T-t)}(F - K)$$

where $d_1 = \frac{\ln \frac{F}{K} + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = d_1 - \sigma\sqrt{T-t}$

Options pricing (cont.)

- If call is deep out-of-the-money
  - $F < K$ and the ratio F/K is very small
  - d_1 and d_2 are also small numbers
- Hence, $N(d_1)$ and $N(d_2)$ will be 0
- Call premium will be zero

$$C = \underbrace{e^{-r(T-t)}}_{\text{discount term}} \underbrace{[\underbrace{F}_{\text{futures price}} \cdot \underbrace{N(d_1)}_{=0} - \underbrace{K}_{\text{strike price}} \cdot \underbrace{N(d_2)}_{=0}]}_{\text{intrinsic value}}$$

$$= e^{-r(T-t)} \cdot 0 = 0$$

where $d_1 = \frac{\ln \frac{F}{K} + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = d_1 - \sigma\sqrt{T-t}$

Options pricing (cont.)

- Black's model: put premium

$$P = \underbrace{e^{-r(T-t)}}_{\text{discount term}} \underbrace{\left[\underbrace{K}_{\text{strike price}} \cdot \underbrace{N(-d_2)}_{\text{CDF of } -d_2} - \underbrace{F}_{\text{futures price}} \cdot \underbrace{N(-d_1)}_{\text{CDF of } -d_1} \right]}_{\text{intrinsic value}}$$

where $d_1 = \frac{\ln \frac{F}{K} + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}}$, $d_2 = d_1 - \sigma\sqrt{T-t}$

Options pricing (cont.)

- Steps to calculate option premiums
  - estimate volatility
  - calculate d_1 and d_2
  - determine $N(d_1)$, $N(d_2)$, $N(-d_1)$, and $N(-d_2)$
  - calculate the call and put premiums

Options pricing (cont.)

- Some important points
  - If we consider futures, calls and puts, combinations of two of them can be used to trade a synthetic position in the third
  - Synthetic futures (options) positions duplicates gains/losses from a futures (options) contract, but consists of positions in other markets
  - Put-call parity is derived from an arbitrage relationship between calls and puts with the same strike price
  - If the put-call parity does not hold, there are profitable arbitrage opportunities; as these arbitrage opportunities are exploited, put-call parity is reestablished
  - Premiums of European options on futures are determined by intrinsic value, volatility, time to maturity and interest rate based on Black's pricing formula

Volatility

- Recall that parameters of Black's pricing model include
  - strike price
  - underlying futures price
  - interest rate (assumed constant)
  - time to maturity
  - **expected** volatility of futures price
- Expected volatility of the underlying futures price is the only unknown parameter
  - expected volatility is not directly observed in the market

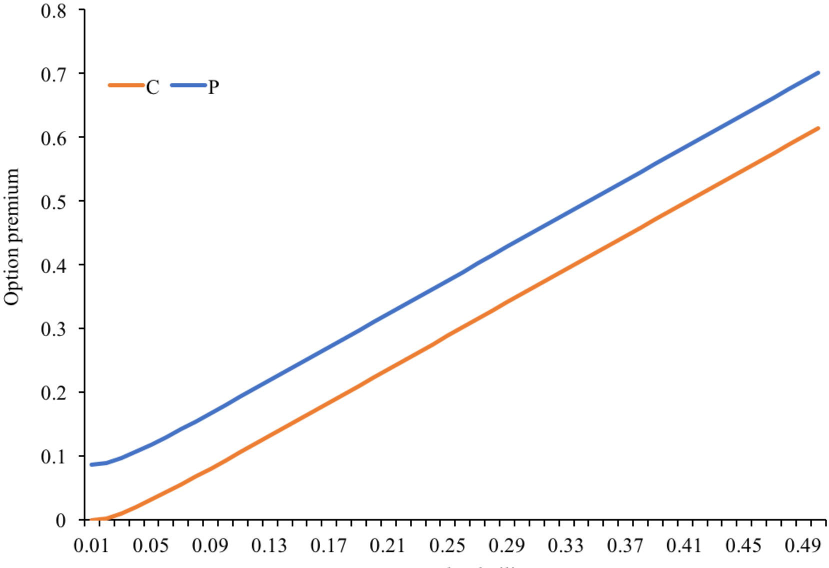
Volatility (cont.)

- So, you need to estimate expected volatility in order to calculate option premiums using Black's model
- Why is expected volatility important?
  - volatility refers to the degree of variability in price changes
  - it reflects the potential of the option to move into-the-money, making it profitable
    (volatility and time to maturity jointly determine the probability of a profitable move)

Volatility (cont.)

- Example: using Black's pricing model for Dec'18 corn on 03/26/2018
  - futures price = \$4.01/bu
  - strike = \$4.10/bu
  - annual interest rate = 5%
  - time to maturity = 264 days $\approx$ 0.72 years
- Estimate call and put premiums for different levels of expected volatility

Volatility (cont.)



Volatility (cont.)

- Two most common methods to estimate expected volatility
  - historical volatility
  - implied volatility
- Historical volatility: use the standard deviation of historical price series to estimate expected volatility
- For example, if you have daily prices for corn since 2004
  - your historical volatility for corn will be the annualized standard deviation of daily **returns** during this period
  - use this value as your expected volatility
- Assumption
  - past volatility is a good predictor of future volatility

Volatility (cont.)

- Historical volatility
  - Potential problems
    - market conditions may change and hence past volatility may not be a good predictor of future volatility
  - Important issues
    - select historical time period that reflects conditions similar to those expected in the future
    - length of the historical price series
    - data frequency (daily, weekly, monthly)
    - whether or not to use weights

Volatility (cont.)

- Historical volatility: numerical example
- Estimate expected volatility for corn prices based on Jan-Sep 2017 dataset

	using daily data	using weekly data
Jan-Sep	0.19	0.17
Mar-Sep	0.20	0.18
May-Sep	0.24	0.21
Jul-Sep	0.27	0.17

Volatility (cont.)

- Implied volatility: estimate of expected volatility derived from actual option premiums
- In Black's pricing model
  - all parameters are known, except premium and volatility
  - put observed premium into the formula and solve it for volatility (instead of solving for the premium)
  - estimated volatility is the volatility implied by the observed premiums traded in the market

Volatilityy (cont.)

- Implied volatility reflects market expectation for future volatility
- How to trade based on implied volatility?
  - compare implied volatility with your own expectations
  - if you believe that implied volatility is too high, option is overpriced and you should go short
  - if you believe that implied volatility is too small, option is underpriced and you should go long
- Implied volatility: potential problems
  - implied volatility can be different from your own expectations because the option is mispriced and/or your expectations are wrong
  - there are several options for the same futures contract maturity, and each option can yield different implied volatilities

Volatility (cont.)

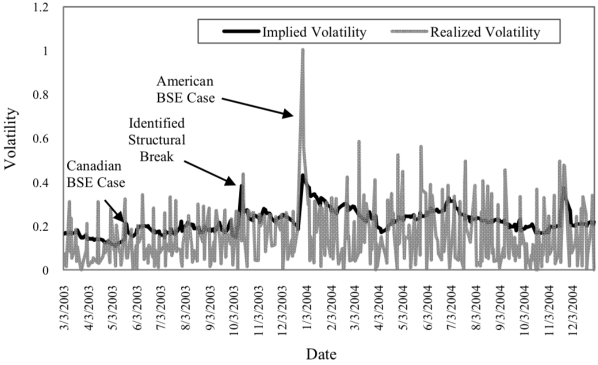


Figure 1. Live cattle daily implied and realized volatility, 3/2003–12/2004

Brittain, Garcia, and Irwin (2011, *JARE*)